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Acoustical Measurements by Time Delay Spectrometry*

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A new acoustical measurement technique has been developed that provides a solution for the conflicting requirements of anechoic spectral measurements in the presence of a reverberant environment. This technique, called time delay spectrometry, recognizes that a system-forcing function linearly relating frequency with time provides spatial discrimination of signals of variable path length when perceived by a frequency-tracking spectrum analyzer.

INTRODUCTION It is a credit to technical perseverance that the electronic subsystems which make up an audio installation have been brought to a high state of perfection. It is possible not only to predict the theoretical performance of a perfect electronic subsystem, but to measure the deviation of the performance of an existing subsystem from that perfect goal. The capability of measuring performance against an ideal model and of predicting the outcome for arbitrary signals is taken for granted. Yet the very acoustical signals which are both the source and product of our labors seldom have sufficient analysis to predict performance with comparable analytical validity. The measurement of even the simpler parameters in an actual acoustical system may be laborious at best. There exists, in fact, very little instrumentation for fundamental measurements which will allow prediction of performance under random stimuli. It is the intent of this paper to describe a new acoustical measurement technique which allows "on-location" measurement of many acoustical properties that normally require the use of anechoic facilities. A new acoustical model of a room is also introduced as a natural by-product of this technique. With this model substantial objects may be selectively analyzed for their effect on sound in the room.

The acoustic testing process to be described relies heavily on electronic circuit techniques which may not be familiar to acousticians. As a brief review of fundamental principles it will be recalled that in linear electronic circuit analysis the concept of superposition permits complete analytical description of circuit response under the influence of any driving function which is describable as a distribution of sinusoids. Each sinusoid in the distribution will possess a unique amplitude and time rate of change of angle or frequency, and will produce a network response which is in no way dependent on the existence of any other sinusoid. The response of a network at any particular frequency is therefore obtained quite simply by feeding in a sinusoid of the desired frequency and comparing the phase and amplitude of the output of a network with its input. The response of a network to all frequencies in a distribution will then be the linear superposition of the network response to each frequency. The response of a network to all possible sinusoids of constant amplitude and phase is called the frequency response of that network. The frequency response is in effect the spectrum of frequency distribution of network response to a normalized input. Cascading of linear networks will involve complex multiplication of the frequency response of each included network to obtain an overall response. An analytical solution to such a combination may then be readily obtained from this overall frequency response.

The equivalent frequency response of an acoustical system should, in principle, be the comparable value in analyzing resultant performance. The processes of sound generation, reflection, transmission, and absorption all have their counterparts in network theory. It is well known, however, that any attempt at utilizing a simple sinusoid driving function on a real-world acoustical system will lead to more confusion than insight. Any object with dimensions comparable to a spatial wavelength of the sinusoid signal will react to the signal and become an undesired partner in the experiment. Furthermore, since the velocity of sound in the various media prevents instantaneous communication, time enters into the measurement in the form of standing wave patterns which will be different for each applied frequency. For those acoustical subsystems for which a single frequency response might be meaningful, such as loudspeakers, microphones, or certain acoustical surfaces, special (and expensive) anechoic test areas are utilized in an attempt

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to remove acoustically any object that might interfere with the measurement. Unfortunately there are many acoustical situations for which such a measurement appears to be impossible or even meaningless. A single frequency response of an auditorium, for example, will be of no use in evaluating the "sound" of the auditorium as perceived by an observer. The large number of reflecting surfaces give a time-of-arrival pattern to any attempted steady-state measurement which prevents analytical prediction of response to time-varying sound sources. Clearly an auditorium has not a single frequency response, or spectral signature, but rather a linear superposition of a large number of spectral responses, each possessing a different time of arrival.

Here we have the essence of many real-world acoustical measurement problems. It is not a single frequency response that must be measured but a multiplicity of responses each of which possesses a different time delay. The room in which an acoustical measurement is made simply adds its own series of spectral responses, which may mask the desired measurement. Selection of the proper responses will yield a set completely defining the acoustical system under test so long as superposition is valid. Conversely, once the entire set of spectra are known along with the time delay for each spectrum it should be analytically possible to characterize the acoustical system for any applied stimulus. Traditional steady-state techniques of measurement are unable to separate the spectra present in a normal environment since a signal response due to a reflection off a surface differs in character from a more direct response only in the time of arrival following a deliberately injected transient. In the discussion to follow a method of time-delay spectrometry will be developed which allows separation and measurement of any particular spectral response of an acoustic system possessing a multiplicity of time-dependent spectral responses. A practical implementation of this technique will be outlined using presently available instruments. Analytical verification of the technique will be developed and a discussion included on acoustical measurements now made possible by this technique.

EVOLUTION OF MEASUREMENT TECHNIQUE

In evolving the concept of time-delay spectrometry it will be instructive first to consider a very simple measurement and then to progress by intuitive reasoning to the general case. Assume that it is desired to obtain the free-field response of a loudspeaker situated in a known reverberant environment. A calibrated microphone will be placed at a convenient distance from the speaker in the direction of the desired response. With the acoustical environment initially quiescent, let the speaker suddenly be energized by a sinusoid signal. As the speaker activates the air a pressure wavefront will propagate outward at a constant velocity. This pressure wavefront will of course not only travel toward the microphone but also in all other directions with more or less energy. Assume that the microphone is connected through a relatively narrow-bandwidth filter to an indicating device, and that this filter furthermore is tuned to the exact frequency sent to the loudspeaker. As the leading edge of the pressure wave passes the microphone, the only contributor to this wave could be the loudspeaker since all other paths from reflective surfaces to the microphone are longer than the direct path. The loudspeaker will take some time to build up to its "steady-state" excitation value and the microphone and tuned circuit will similarly have a time constant. If the system arrives at a steady-state value before the first reflected sound arrives at the microphone, this steady-state is in effect a free-field measurement at the frequency of the impressed sinewave. Note that because of the broad spectrum of a suddenly applied sinewave, a tuned filter circuit is necessary to prevent shock-excited speaker or microphone resonances from confusing the desired signal.

The measurement thus described is quite simple and has actually been used by some investigators.[1-3] As long as a fixed filter is used, the measurement must be terminated prior to receipt of the first reflected "false" signal and the system must be de-energized prior to a subsequent measurement. Suppose, however, that the fixed-frequency sinewave is applied to the speaker only long enough to give a steady-state reading prior to the first false signal, then suddenly shifted to a new frequency outside the filter bandwidth. Assume also that by appropriate switching logic, a filter tuned to this new frequency is inserted after the microphone at the precise time that the sound wave *perceived by the microphone* changes frequency. The microphone circuit will thus be tuned to this new frequency and the later reflected false signals of the first frequency will not be able to pass through the new filter. If one continues this process through the desired spectrum it is apparent that the indicator circuit will never "know" that the measurement was performed in a reverberant environment and a legitimate frequency response may thus be measured.

The practical economics of inserting fixed filters and waiting for the starting transient at each frequency to die down weigh heavily against such a system, so an alternative may be considered. Project a smooth glide tone to the speaker and utilize a continuous tracking filter after the microphone. If the tracking filter is tuned to the frequency of the emitted glide tone as perceived by the microphone and if the glide tone has moved in frequency by at least the bandwidth of the tracking filter before the first reflected signal is perceived, no buildup transient is encountered and the measurement will be anechoic even though performed in a reverberant environment. The nature of the glide tone may readily be ascertained by intuitive reasoning. If there is no relative motion between speaker and microphone it can be stated with absolute certainty that the time delay between speaker and microphone is a constant. There is, in other words, a unique and linear relationship between time and distance traveled by the pressure wave. Each reflecting surface will appear to be a new sound source with a time delay corresponding to path length. If we specify that the sweeping tone and the tracking filter combination be capable of maximizing response for all frequencies from any given apparent source then we have required an equivocation of frequency, room spacing, and time. The glide tone satisfying this requirement possesses a constant slope of frequency versus time. If all reverberant energy due to any given frequency has died to an acceptable level a fixed time following excitation, say T seconds, then the glide tone may be allowed to repeat its linear sweep in a sawtooth fashion with a period of no less than T seconds.

While we began by postulating direct loudspeaker measurement it is apparent that we could by suitable choice of sweep rate, bandwidth, and time delay, "tune" in on at least first-generation reflections with the selective exclusion of others, even the direct loudspeaker response. Because the output of the tracking filter yields the spectral signature of the perceived signal with a frequency proportional to time and since selective spatial isolation of the desired signal is obtained by utilizing the fixed time delay between source and microphone, the rationale of the name *time-delay spectrometry* becomes apparent.

Some simple relationships may be directly derived from the basic geometry of a practical situation. Consider the representation of Fig. 1 in which a microphone is connected to a tracking filter "tuned" to perceive a source at a distance X on a direct path. The filter has a bandwidth B Hz; it is seen that the sweep tone will traverse some ΔX in space while within a band B of any given frequency. Define ΔX as the region in space, along the direction of propagation of the acoustic signal, within which the selected signal power will be no less than half the maximum selected value. This is the spatial analog of the half-power bandwidth B of the tracking filter, and will therefore be referred to as the space-equivalent bandwidth. This space-equivalent bandwidth ΔX is related to the tracking filter electrical bandwidth B, the velocity of sound c and the rate of change of frequency $\Delta F/\Delta t$, by

$$\Delta X = B[c/(\Delta F/\Delta t)] \approx c/B. \qquad (1)$$

The last relation is based upon an optimized bandwidth which is the square root of the sweep rate. While not immediately obvious, this optimized bandwidth is common in sweeping analyzers of the variety recommended for this measurement.[4]

The signal perceived by the microphone is that emitted by the speaker some time in the past. To visualize the relationship consider Fig. 2 diagramming the behavior of a sweep tone repetitive in a time T. The signal emitted from the source or transmitter will be denoted by F_t while that received by the microphone is F_r. It is usually desired to sweep through zero frequency; this is shown in the diagram as a signal dropping in frequency uni-

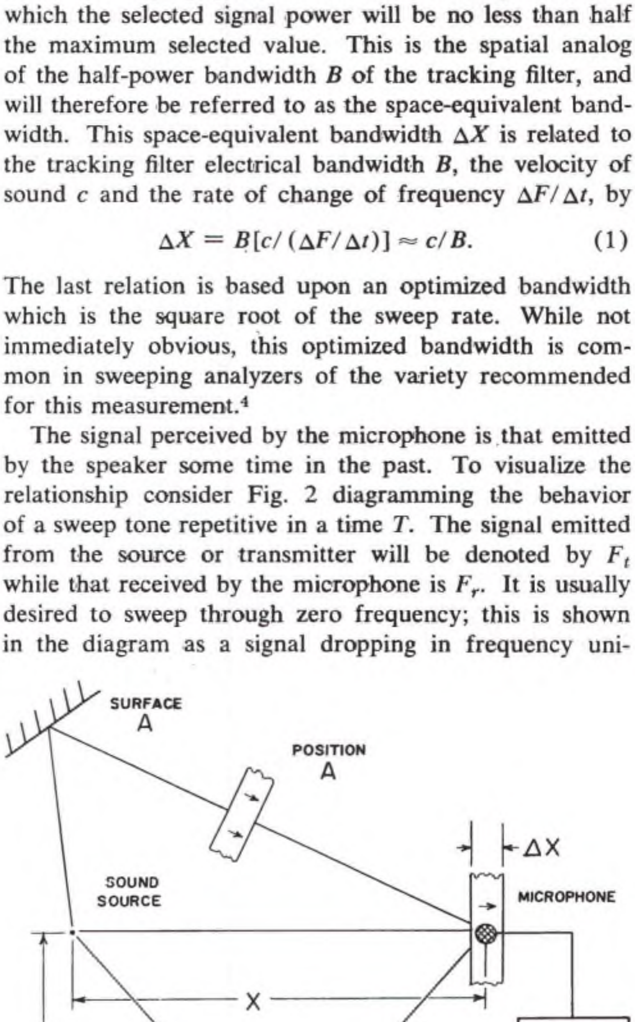


Fig. 1. Positional representation of direct and reflected acoustic pressure waves of constant frequency and fixed space-equivalent bandwidth as emitted by a swept frequency source and perceived by a microphone connected to a tracking filter tuned to maximize the response at a distance X.

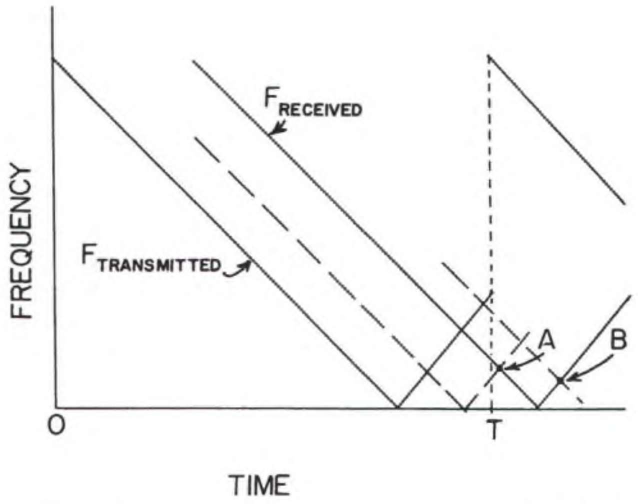


Fig. 2. Frequency plot of the sweep tone passing through zero frequency, showing room signals received as a function of time.

formly with time until zero frequency and then rising again. We are thus going through zero beat. In this diagram the direct signal possessing the shortest time delay is shown dashed. It is clear that the relationship between distance, transmitted and received frequencies at any instant, and sweep rate is

$$X = (F_r - F_t)[c/(\Delta F/\Delta t)]. \qquad (2)$$

This states the fact that to "tune" to the response of a signal X feet away from the microphone it is only necessary to offset the frequencies of glide tone and tracking filter by a fixed difference. Referring again to Fig. 1 we could check the response of the reflective surface A in one of two ways. First, we might offset the source and received frequencies to yield the primary sound signal as shown and then, without changing this frequency offset, physically transport the microphone to position A. It is assumed that a position may be found for which no contour of undesired reflected or direct sound comes into space tune at position A. The second way of adjusting for surface A would be to maintain the existing microphone position and change the offset between transmitted and received frequencies to account for the longer path of A. Both techniques allow for probing the effective acoustic surfaces in the room. The room is, of course, filled with sound, but since we have an instrument uniquely relating time, space, and frequency we have in effect "frozen" the space contours of reflected sound. We may probe in space merely by adjusting a frequency offset. Analytically we may state that we have effected a coordinate conversion which trades spatial offset for driving frequency offset, but which retains intact all standard acoustical properties including those due to the frequency of the system-driving function. This is the power of this technique, for so long as the acoustic properties remain substantially linear, a hopeless signal combination in normal spatial coordinates transforms to a frequency coordinate generally more tractable to analyze.

In considering the uniqueness of signals perceived by frequency offsetting it may be observed that a transmitted signal passing through zero frequency will reverse phase at the zero frequency point and continue as a real frequency sweep with reversed slope of frequency *vs* time.

Thus, near zero there will be two transmissions of a given frequency during one sweep. The signals corresponding to the same frequency slope as that of the tracking filter will be accepted or rejected in total as the proper offset is entered for the appropriate time delay. The duplicate frequencies near zero possessing a different slope will cause a "ghost" impulse to appear when the instantaneous frequency perceived by the microphone corresponds to that of the tracking filter, regardless of the offset. Similarly, a tracking filter may be set to "look" for frequencies on both sides of zero. If the signal under analysis is a first-generation reflection, the main signal from the speaker will arrive sooner than this reflection and an impulse will occur at position *A* in Fig. 2. If, on the other hand, we are looking for a signal prior to the main signal, the impulse might appear at position *B*. Thus a repetitive sweep passing through zero frequency may contain image impulses due to signal paths other than that to which the tracking filter has been tuned. All sweeps with sufficiently long periods which do not pass through zero frequency will avoid any such ambiguities.

Another relationship worthy of consideration is the distance one may separate speaker and microphone without interference from a large object offset from the direct line of sight. Such considerations arise when measurements are made near floors, walls and the like. Surprisingly, an off-center reflecting object such as a floor limits the maximum distance of speaker-microphone separation. Consider Fig. 1 where the speaker and microphone are assumed to be a height h off the floor. The maximum distance will be that for which the path length from the image speaker is just equal to $X+(\Delta X/2)$. The maximum usable line-of-sight distance between speaker and microphone, X_{max}, is related to height h and space equivalent bandwidth ΔX by

$$X_{max} = (\Delta X/4)\{[h/(\Delta X/4)]^2-1\}. \qquad (3)$$

Note that if it is necessary to make lower frequency measurements or, what is the same thing, higher definition measurements, the distances scale up to values comparable to the wavelength of that frequency for which the period is the rise time of the tracking filter. This follows from observing that Eq. (1) may be rewritten as

$$\Delta X \cdot B \approx c \qquad (4)$$

PRACTICAL IMPLEMENTATION

While it might appear at first glance that the acoustical measurements outlined in the previous paragraph require specialized apparatus, equipment necessary to perform the described measurement is readily assembled from commercially available instruments. The tracking filter is a portion of an audio spectrum analyzer. This instrument is a narrow-band superheterodyne receiver tuned through the audio spectrum with a local oscillator swept linearly in time—the needed frequency characteristic. Depending upon the resolution required, the commercially available analyzers will sweep through a variety of given frequency dispersions in a repetitive sawtooth fashion. A sweep rate of one per second is typical for spectrum analyzers covering the full audio spectrum. The output of this audio tracking filter is rectified and applied to the vertical axis of a self-contained oscilloscope with the linear time axis swept horizontally. Because of the repetitive nature of the display, this instrument provides a visual presentation of signal energy *vs* frequency. The bandwidth of the spectrum analyzer is usually chosen to be the narrowest possible without losing information when swept past a complex spectrum.

The sweep tone for driving the loudspeaker may be obtained by down-converting the spectrum analyzer local oscillator to the audio band. If the local oscillator is heterodyned with another oscillator equal to the analyzer intermediate frequency, the difference frequency will always be at the precise frequency to which the analyzer is tuned. The proper offsetting frequency for spatial tuning of acoustical signals is obtained by detuning the fixed oscillator from the intermediate frequency. A down-converting synchronously sweeping generator is usually available as an accessory to the spectrum analyzer. To perform time-delay spectrometry it is only necessary to substitute a stable tunable oscillator for the fixed crystal oscillator. Where extreme accuracy is required it may be necessary to use a frequency counter to monitor the proper offset frequency. The remainder of the equipment consists of the usual power amplifier, microphone, and preamplifier.

Figure 3 diagrams a complete setup capable of performing any or all of the measurements outlined. While the cost of such assembled equipment is greater than

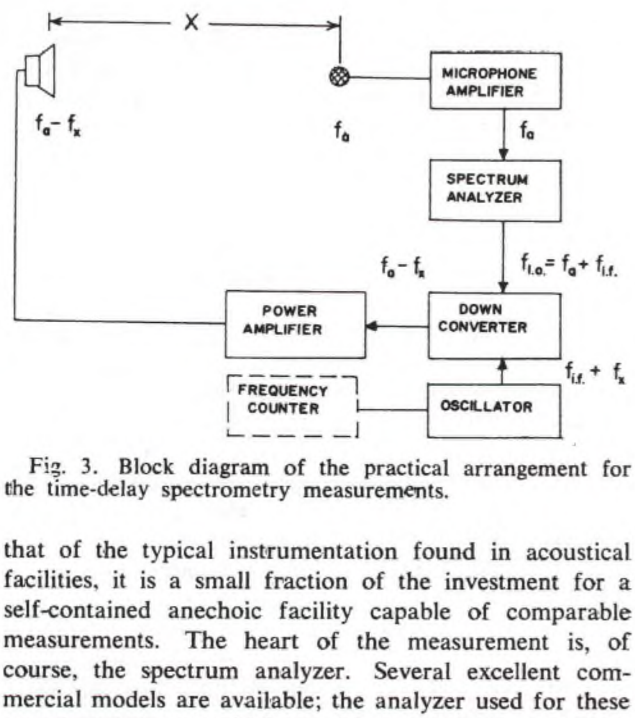


Fig. 3. Block diagram of the practical arrangement for the time-delay spectrometry measurements.

that of the typical instrumentation found in acoustical facilities, it is a small fraction of the investment for a self-contained anechoic facility capable of comparable measurements. The heart of the measurement is, of course, the spectrum analyzer. Several excellent commercial models are available; the analyzer used for these experiments is one which provides a continuously adjustable sweep width from 200 Hz to 20 KHz with a sweep center frequency separately adjustable from dc to 100 KHz. The bandwidth is tracked with the sweep width control to yield optimum resolution at the sweep rate of one per second. According to Eq. (1), this means that at maximum dispersion a 20 KHz spectrum may be obtained at 141 Hz resolution at 7.8 ft space-equivalent bandwidth. If measurements are desired valid to 32 Hz, for example, the sweep width would be set to 1000 Hz and the space-equivalent bandwith would be 34 ft. Where smaller space-equivalent bandwidths are desired and the decreased resolution can be tolerated, it is possible to increase the internal analyzer sweep rate by as much as a factor of five by minor circuit modification without compromising the validity of the measurement.

APPLICATIONS OF TIME DELAY SPECTROMETRY

Perhaps the best way of illustrating the use of time-delay spectrometry would be a detailed look at a typical measurement. Assume that an on-axis pressure response frequency characteristic is desired for a direct radiator loudspeaker. The entire spectrum from zero to 20 KHz is desired, with a frequency resolution of 140 Hz acceptable. The room selected for measurement has an 8 ft floor-to-ceiling height and a reverberation time of less than one second.

The space-equivalent bandwidth is approximately 7.8 ft (from Eq. 1), which means that no substantial object should be placed within 4 ft of the pickup microphone. An acceptable sweep rate is one 20 KHz sweep per second. From Eq. (3) it can be seen that if the speaker is placed halfway between floor and ceiling, the maximum distance the microphone should be placed from the speaker is six feet. This assumes, of course, a worst-case specular reflection from both floor and ceiling as well as a uniform polar response from the speaker.

The speaker under test is then placed 4 ft off the floor and more than 4 ft from any substantial object. The pickup microphone should be placed on-axis within 6 ft of the speaker and at least 4 ft from any reflective surface. The microphone is electrically connected to a suitable preamplifier which in turn feeds into the spectrum analyzer input. The down-converted local oscillator signal from the tracking oscillator is fed to an appropriate power amplifier driving the speaker. The offset oscillator which replaces the crystal in the tracking oscillator is connected to a frequency counter to complete the electrical setup.

With the deviation and deviation rate set to the desired test limits in the spectrum analyzer, a sweeping tone will be heard from the speaker. If the microphone-to-speaker distance is known, the offset oscillator should be set in accordance with Eq. (2) to achieve maximum deflection of the spectrum analyzer display. For example, a 6 ft separation will require a 114 Hz offset. It is at this point that the advantage of a tracking analyzer is evident: not only may the intensity of sound from the speaker be modest, but also it is not necessary to cease all sound and motion in the room while measurement is in progress. Any reasonable extraneous sound may be tolerated, and mobility need only be restricted to the extent that travel between loudspeaker and microphone is discouraged.

The spectrum analyzer display should remain stationary in vertical deflection. To ascertain that the proper frequency offset is used, the offset oscillator may be detuned on both the high and low side; the resultant display should show a reduction in amplitude. The peaked stationary pattern on the screen of the spectrum analyzer will be a plot of pressure response *vs* frequency with a smoothing bandwidth of 0.7% of the displayed dispersion.

Without altering the test setup several types of acoustical measurements may be made. Off-axis response of the speaker may be obtained by rotating the speaker by the required angle and immediately viewing the results on the analyzer. If it is desired to determine the sound transmission characteristic of a particular material, for example, one first obtains the on-axis speaker response and then interposes a 4 ft sample between speaker and microphone, noting the new response. If the spectrum analyzer is set for logarithmic deflection, the sound absorption in decibels is the difference in the two responses independent of speaker or microphone response.

The reflection coefficient may similarly be obtained by positioning the material behind the microphone by at least 4 ft and retuning the offset oscillator for the reflected signal. If the microphone is rotated 180° to remove its polar characteristics from the measurement, and if the proper space loss is entered, the reflection coefficient is obtained directly as a function of frequency, independent of microphone and loudspeaker characteristic. If one does not wish to compute the space loss, and space permits, the microphone may be physically transported to the position of maximum on-axis response with the reflective surface removed, and space loss obtained as response difference from the earlier position.

ROOM SPECTRUM SIGNATURES

The analysis up to this point has been directed at producing an electrical signal which when fed to a speaker will allow two very important measurements to be made on the sound picked up by a microphone. First, only the selected direct or reflected signal will be pulled out and displayed. Second, the spectral response of the selected signal will be directly presented. With this in mind, consideration will now be given to a characterization of sound in a room by time-delayed spectra.

Assume that an observer is positioned in a room with a localized source of sound such as a loudspeaker. This loudspeaker furthermore has a pressure frequency response characteristic, such as Curve A in Fig. 4. As the loudspeaker is energized by any arbitrary signal, the first sound heard by an observer will be characterized by Curve A. (This does not imply that all the frequencies

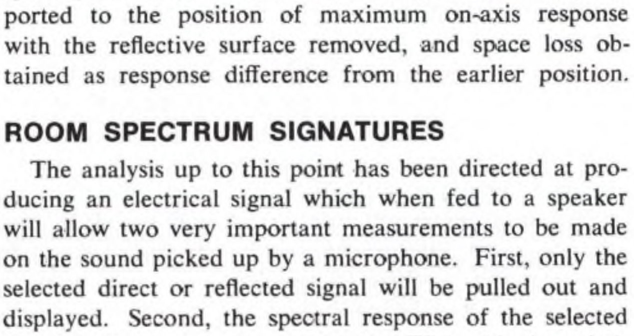


Fig. 4. Acoustic model of room spectrum signatures as perceived by an observer listening to a sound source with spectral distribution *A*.

are present, but only that those electrical signal components which are present are modified in accordance with Curve A.) Since the loudspeaker output varies as a function of time it will be assumed that Curve A is representative of the pressure output at a particular instant in time. We have in effect plucked out that signal which is characteristic of a given instant and will follow this pressure wave as it travels through the room and intercepts the observer.

A few milliseconds after the principal pressure wave A has passed, a second pressure wave B will be perceived by the observer. This is a reflected wave and is made up of the principal wave modified by the frequency response of the reflecting surface. As in electronic circuit theory, the observed resultant frequency response of B is the product of the frequency response of A and the frequency response of the reflecting surface. Beside the obvious time delay introduced by the longer path length, the surface creating the wave B may introduce a dispersive smearing of portions of the frequency range. This may be due to surface irregularities which act as local scattering centers or to acoustic impedance variations causing effective displacement of the position of optical and acoustic surfaces. If the reflecting surface has very little depth variation and is primarily specularly reflective, such as a hard wall or ceiling, the time-pressure profile might appear as wave B. If on the other hand the reflecting object has depth, such as a chair, the time-pressure profile may take on the character of the third pressure wave C. Here the effect is a distinct broadening of the time during which the observer perceives the signal. Perhaps a better name than reflection coefficient in the case of such an object might be scattering coefficient.

As time progresses the observer will perceive more scattering spectra with increasing density and generally lower amplitude until all sound due to the initial loudspeaker excitation has fallen below a predetermined threshold. The complete pressure-frequency-time profile will be a unique sound signature of the room as perceived by an observer listening to the loudspeaker. This is, after all, the way in which the sound reaches the observer. A more general characterization of the room itself would replace the loudspeaker and its unequal polar pressure response by an analytically uniform pressure transducer. Each position of observer and transducer will have its unique pressure-frequency-time profile. It quite frequently happens, however, that such a general characteristic is of little concern and what is desired is the observer-loudspeaker situation of Fig. 4.

The complete pressure-frequency-time profile represented by Fig. 4 is a useful acoustic model of a reverberant room in which an observer perceives a localized source of sound. As far as the observer is concerned there are many apparent sound sources. Each of these has the same "program content" as the primary source of sound but possesses its own spectral energy distribution and unique time delay, corresponding to its apparent location with respect to the observer. The effect of any given object in the acoustic environment may be determined in this model by noting the equivalent time delay from source to object to observer, and analyzing the scattering spectrum corresponding to this delay. The entire set of time-dependent scattering spectra should allow detailed analysis of this system for any time-dependent stimulus applied to localized source of sound.

This acoustic model of a room is seen to tie directly to time-delay spectrometry. The electronic sweep tone fed to the loudspeaker will in effect represent all possible frequencies in the chosen sweep range. The time axis of Fig. 4 could also be labeled offset oscillator frequency; the signal displayed on the spectrum analyzer will be seen to be the frequency spectrum of the selected time delay. The space-equivalent bandwidth, ΔX, will establish the ability to resolve independent scattering spectra since time-delay spectrometry forms an equivocation of time, offset frequency, and distance. It is immediately apparent that the selection of loudspeaker on-axis response used as an example for generation of time-delay spectrometry entails selection of Curve A and is only a special case of a more general acoustical measurement technique. It is possible by suitable choice of loudspeaker and microphone position to "pull out" important acoustic properties of a room without destroying the room or utilizing large-scale digital computer techniques.

Analytically, a surface at a distance corresponding to a time delay of t_k seconds may be characterized by a frequency spectrum multiplied by a linear phase coefficient. The cumulative distribution of these sources is represented by Fig. 4 and may be expressed as

$$R(\omega) = \sum_k S_k(\omega)\, e^{-i\omega t_k}, \tag{5}$$

where $R(\omega)$ is the cumulative distribution of all sources, both real and apparent, as perceived by an observer listening to a localized source of sound in a reverberant environment, and $S_k(\omega)$ is the spectral energy distribution of the source at a distance corresponding to a time delay of t_k seconds. The angular frequency ω, expressed in radians per second, will be used in this paper for analytical simplicity. Since the absolute magnitude of $S_k(\omega)$ is measured by time-delay spectrometry it is not necessary to introduce a space loss term. If the source has a spectral distribution $P(\omega)$, then the observer will perceive a signal which has a frequency spectrum that is the product of $P(\omega)$ and $R(\omega)$. The impulse response of the room, $r(t)$, is simply the Fourier Transform of $R(\omega)$:

$$r(t) = (1/2\pi) \int_{-\infty}^{\infty} R(\omega)\, e^{+i\omega t}\, d\omega \tag{6}$$

and the time-varying signal $o(t)$ perceived by an observer for a program source $p(t)$ is the convolution integral of program source and room response

$$o(t) = \int_{-\infty}^{\infty} p(\tau) r(t-\tau) d\tau \tag{7}$$

The concept expressed by Eqs. (6) and (7) is certainly not new to the field of acoustics. The difficulty in applying these relations lies in the massive amounts of data that must be processed before the effect of a particular object in a room may be evaluated. The acoustic model of Eq. (5) as represented in Fig. 4 leads to a simpler analysis since time-delay spectrometry isolates the $S_k(\omega)$ function of acoustic surfaces and presents the information in a meaningful form which requires no further reduction. While not a cure-all for acoustic analysis, time-delay spectrometry provides a good insight to actual acoustic situations.

EXPERIMENTAL RESULTS

Sufficient experimental evidence has been collected to assure validity of the technique. The examples below give some insight into the nature of the spectrum display.

Loudspeaker Testing

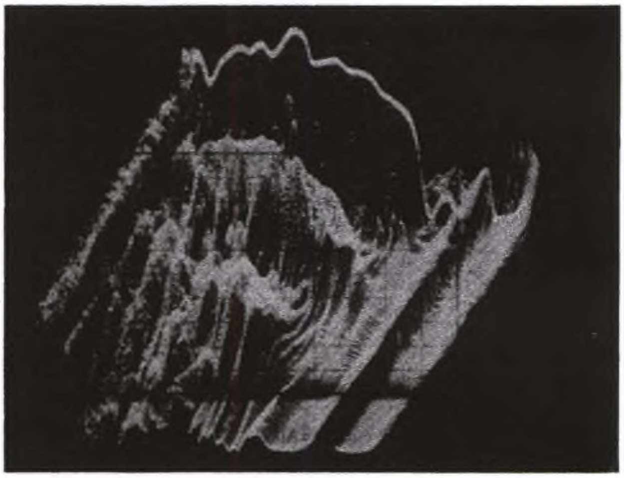
A single-cone 8-in. loudspeaker housed in a ported box was chosen for test, in a room deliberately selected to be a poor environment for loudspeaker testing. The room measures 24 × 10 × 7 ft. The floor is hard vinyl on cement and the ceiling is plaster, while all other surfaces are hard wood. Miscellaneous objects throughout the room reduce the open area so that when the loudspeaker-to-microphone distance is at the calculated maximum of Eq. (3) the smallest separation from the microphone to any substantial object is one half the space-equivalent bandwidth. (The room spacing is chosen to be no larger than the calculated minimum dimension in order that the concept may be better demonstrated.) The spectrum measurement was made to include dc to 10 KHz at a dispersion rate of 20 KHz per second. Fig. 5a is a photograph of the on-axis pressure response in decibels as a function of linear frequency. Since the analyzer sweeps through zero-beat to bring dc at the left-hand edge, a mirror image response is seen for those frequencies to the left of dc.

In order to illustrate the nature of the response to be obtained for the steady-state application of sinewaves, the test setup was left intact and the spectrum analyzer driven at such a slow rate that the room acoustics entered into the measurement. Figure 5b is the result of this measurement. The sweep width, vertical sensitivity, and analyzer bandwidth are identical to those of Fig. 5a, but the sweep rate is such that it takes over three minutes to go from zero frequency to 10 KHz. The camera aperture was optimized to give a readable display, and consequently the large number of standing wave nulls and peaks did not register well. The wild fluctuation in reading as a function of frequency is quite expected and lends credence to the futility of steady-state loudspeaker measurement in the acoustic equivalent of a shower stall.

One certain way of verifying that a "free-field" measurement is made is to impose inverse square law.[5] Figure 5c is a time-delay spectrograph of an inverse square law measurement. The setup of Fig. 5a and 5b was spectrographed and appears as the lower trace in Fig. 5c. The loudspeaker was then moved toward the microphone to one half its former distance, and a second exposure was taken with the proper offset oscillator setting. Since the display is logarithmic, the two traces should be parallel and separated by 6 dB, and indeed this is seen to be the case. The minor discrepancies are due to the fact that the first measurement was made at 5 ft, which is the maximum theoretical separation for the room and dispersion rate. The closer measurement is made at 2½ ft where the loudspeaker does not exactly appear as a point source since the separation-to-diameter ratio is only four.

a.

b.



c.

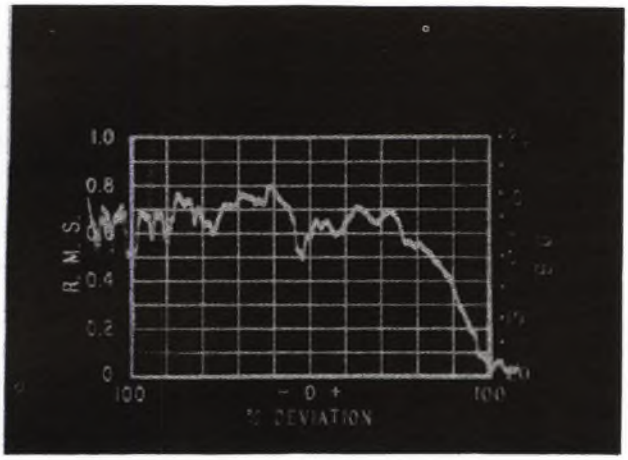
Fig. 5. Oscillographs of logarithmic pressure response measurements on a single-cone loudspeaker from dc to 10 KHz, measured in a reverberant environment by time-delay spectrometry. **a.** Using rapid sweep. **b.** Using slow sinewave sweep. **c.** Showing inverse square response as a function of distance.

Isometric Display of Room Signature

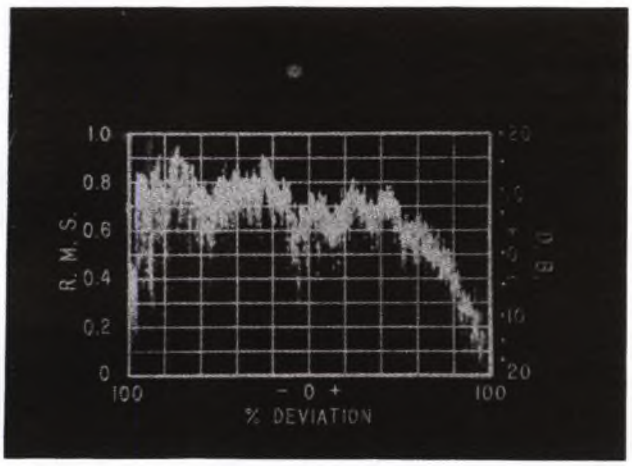
The acoustic model of a room proposed above resulted in a three-dimensional plot of pressure, frequency, and time. Time-delay spectrometry is a means of probing and presenting this model. In order to visualize this phenomenon, the spectrum analyzer display has been modified to accept what may be called Isometric Scan. The offset oscillator is in this instance a linear voltage-controlled oscillator. Each sweep of a time delay spectrograph is presented normally as a plot of pressure *vs* frequency. Isometric scan advances the offset oscillator following each sweep and creates the effect of the third dimension of offset frequency exactly as one illustrates a three-axis system on two-dimensional paper, that is, by a combined horizontal and vertical offset displacement of the sweep. The net effect is a presentation identical in form to Fig. 4.

Figure 6a is an isometric scan of the room and setup of the preceding loudspeaker test. The time axis commences at zero when the principal on-axis signal is received and advances through 43 msec which is equivalent to twice the longest room dimension. All frequencies from zero through 10 KHz are displayed. The exceedingly large number of "alpine peaks" in evidence are only partially visible in the photograph since it is difficult to capture the visual impression an observer sees as this three-dimensional plot unfolds on the oscilloscope screen.

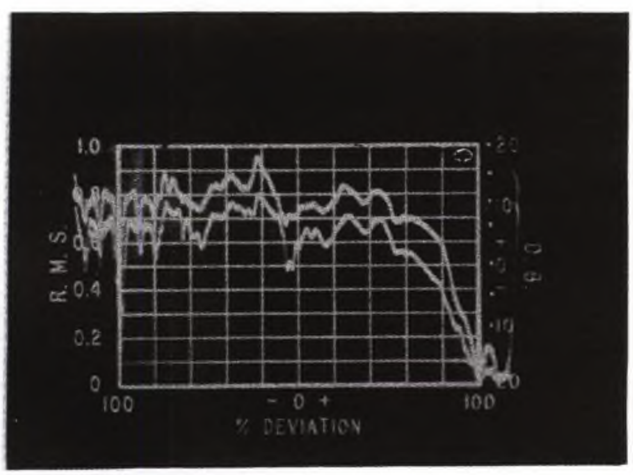
Figure 6b is an isometric scan of a multicellular horn from dc to 20 KHz and is a good representation of the principal pressure wave A of Fig. 4 with the finite space-equivalent bandwidth in this case equal to ½ ft. Figure 6c is an interesting example of multiple reflections. An acoustic suspension loudspeaker system was placed on a floor pointing upward at a hard ceiling, with an omnidirectional microphone halfway between speaker and ceiling. The sweep extends from dc to 20 KHz with a 20 KHz marker "fence" in evidence in the spectrograph. The space-equivalent bandwidth is ½ ft. Time starts at the receipt of the principal wave. The second peak corresponds to the ceiling reflection. The third peak is the reflection of the ceiling wave off the loudspeaker itself. Some of the energy is reflected by the loudspeaker grille and some by the speaker cones themselves yielding beautiful doppler data, and some energy penetrates through the loudspeaker to the floor.



a.



b.



c.

Fig. 6. Oscillographs of isometric displays of spectral signatures using time-delay spectrometry. **a.** Room signatures of the setup of Fig. 5. **b.** Principal response of a multicellular horn from dc to 20 KHz. **c.** Spectral signatures obtained with loudspeaker system pointed at ceiling, including direct response, ceiling reflection, and speaker reflection of ceiling wave.

ANALYTICAL VERIFICATION

The measurement technique of the previous paragraphs was developed in a highly intuitive manner. Although experimental measurements tend to verify the technique, it is still necessary to analyze two very important considerations: first, the nature of a repetitive glide tone, and second, the validity of identifying the spectrum analyzer display with the acoustic spectrum.

Fourier Spectrum of a Repetitive Linear Glide Tone

The system-forcing function developed for time-delay spectrometry has the characteristic shown in Fig. 7. Loosely speaking, it is a signal which has an instantaneous frequency linearly proportional to time for a total period of T seconds and then repeats the cycle indefinitely at this period. Clearly the signal must have a Fourier series spectrum of terms with periods which are integral submultiples of T. It cannot, in other words, be a continuum of frequencies. Yet the intuitive development presupposed a forcing function which not only did not have "holes" in the spectrum but also was of constant amplitude.

Readers familiar with frequency modulation will recognize that the signal of Fig. 7 is in reality a linear sawtooth frequency modulating a carrier halfway between the peak deviation frequencies. Let the maximum and minimum deviation frequencies be ω_2 and ω_1 respectively and the period of sweep T seconds. The function to be defined then has an instantaneous frequency ω_{inst} given by

$$\omega_{inst} = \left(\frac{\omega_2 - \omega_1}{T}\right)t + \left(\frac{\omega_2 + \omega_1}{2}\right) \triangleq \frac{D}{T}t + \omega_c \quad (8)$$

where the angular dispersion D and effective carrier frequency ω_c are introduced for simplicity. Since we are interested in time-dependent factors, the time phase dependence becomes

$$\phi(t) = \int_0^t \omega_{inst} dt = (D/2T)t^2 + \omega_c t. \quad (9)$$

The actual function of time which is to be expanded

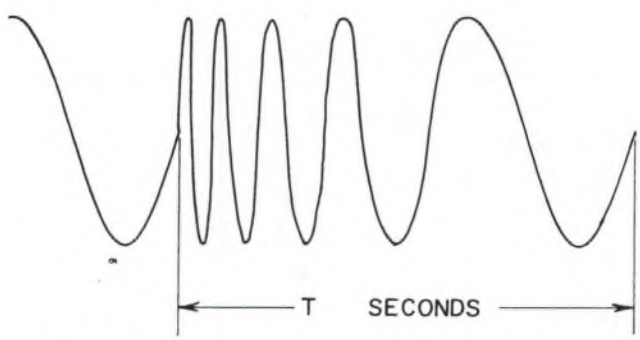


Fig. 7. Graphic representation of the repetitive sweep tone used as system-forcing function.

in a Fourier series is that signal possessing the phase $\phi(t)$, or

$$E(t) = \cos\phi(t) = \tfrac{1}{2}[e^{i\phi(t)} + e^{-i\phi(t)}]. \qquad (10)$$

From an analytical standpoint it is simpler to use the exponential form. Since each complex exponential is separable into the product of a steady-state term and a term periodic in T, it is sufficient to expand this periodic term in a Fourier series. Furthermore, the positive frequency portion will be analyzed first and the negative frequency terms considered from this; thus the Fourier frequencies and coefficients become, in exponential form:

$$e^{i(D/2T)t^2} = \sum_{N=-\infty}^{\infty} C_N e^{iN\omega_0 t} \qquad (11)$$

$$C_N = (1/T)\int_{-T/2}^{T/2} e^{i(D/2T)t^2} e^{-iN\omega_0 t}\,dt \qquad (12)$$

where

$$\omega_0 = 2\pi/T.$$

By completing the square, multiplying and then dividing by a normalizing factor $(2/\pi)^{1/2}$, C_N becomes

$$C_N = \qquad (13)$$
$$\frac{1}{T}\left(\frac{\pi T}{D}\right)^{1/2} e^{-i(T/2D)(N\omega_0)^2}\left(\frac{2}{\pi}\right)^{1/2}\int_{-T/2}^{T/2} e^{ix^2}dx$$

where

$$x = (D/2T)^{1/2}[t-(T/D)N\omega_0]$$

or, by noting Eq. (8),

$$C_N = \frac{1}{T}\left(\frac{\pi T}{D}\right)^{1/2} e^{-i(T/2D)(N\omega_0)^2} \qquad (14)$$
$$\left[\left(\frac{2}{\pi}\right)^{1/2}\int_0^{\omega_2} e^{ix^2}dx - \left(\frac{2}{\pi}\right)^{1/2}\int_0^{\omega_1} e^{ix^2}dx\right]$$
$$x = (T/2D)^{1/2}[\omega_{inst} - (\omega_c + N\omega_0)].$$

Normally analysis would stop here because the definite integrals can be evaluated only as an infinite series. However, the complex expansion of the exponential integral

$$\left(\frac{2}{\pi}\right)^{1/2}\int_0^u e^{ix^2}dx = C(u) + iS(u) \qquad (15)$$

is made up of the Fresnel cosine integral $C(u)$ and Fresnel sine integral $S(u)$, both of which have been tabulated.[6]

Noting the fact that the argument changes sign in the second integral, we can write the resultant Fourier coefficient for the N^{th} sideband term as

$$C_N = (1/T)(\pi T/D)^{1/2} e^{-i(T/2D)(N\omega_0)^2} \qquad (16)$$
$$[C(\omega_1) + C(\omega_2) + iS(\omega_1) + iS(\omega_2)].$$

The C_n's are the coefficients of the Fourier components or, considered another way, are the sideband terms and consist of the product of the quantized spectrum of a continuous swept tone and a complex Fresnel integral modifying term involving the finite frequency terminations. Figure 8 illustrates how to calculate the coefficient for the N^{th} sideband; Fig. 9 is a plot of the locus of the complex value of the Fresnel integral term for a high deviation ratio, which is common for time-delay spectrometry. Note that the spectrum is completely symmetrical about the effective carrier and that while the dropoff is shown for one band edge, the other band edge is identical in form. It is seen that the magnitude is essentially constant throughout the spectrum, as expected, and that within the swept band the phase departure is limited to 15° and rapidly approaches 0° at center frequency.

The function

$$e^{ix^2} \qquad (17)$$

goes through a first zero crossing for the real value, where $u = 1$ when

$$x^2 = \pi/2. \qquad (18)$$

From Eq. (14), this occurs when

$$\Delta\omega = (2D/T)^{1/2}(\pi/2)^{1/2} = (\pi D/T)^{1/2}. \qquad (19)$$

Since the relation between optimized bandwidth B and dispersion rate D/T for the analyzer is

$$B \cong (D/2\pi T)^{1/2} = [1/(2\pi)^{1/2}](D/T)^{1/2}, \qquad (20)$$

then

$$\Delta\omega \cong B/\sqrt{2}. \qquad (21)$$

In other words, even though Fig. 9 shows an overshoot at band edge of diminishing amplitude and increasing frequency as one approaches the effective carrier, these ripples are not distinguishable to a tracking analyzer since they are smoothed by the proper filter bandwidth and the analyzer cannot distinguish this distribution from a true

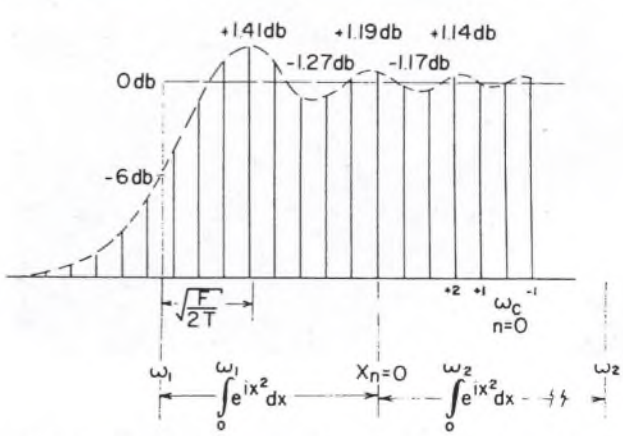


Fig. 8. Method of calculating sideband coefficients for linear sawtooth frequency-modulated carrier with peak angular frequencies ω_1 and ω_2, dispersion $D = 2\pi F = \omega_2 - \omega_1$, and sweep time T.

constant amplitude. Thus the desire for an effective constant amplitude continuum is satisfied.

While the conventional rates of time-delay spectrometry dictate a high effective modulation index yielding the near band-edge distribution of Fig. 9, it may be of some interest to investigators to determine the distribution for low effective indexes. This may be done quite simply by noting that a complex plot of Eq. (15) yields a Cornu spiral. This is shown in Jahnke and Emde[6] and appears

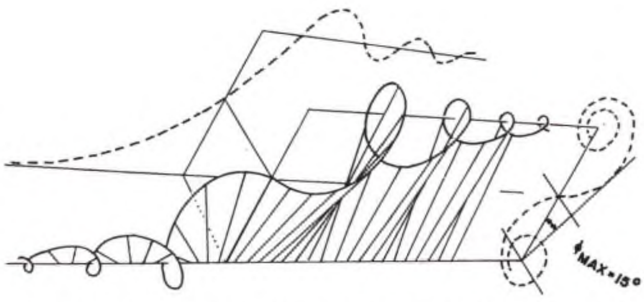


Fig. 9. Locus of the complex value, including magnitude and phase, of the Fresnel Integral term in the coefficient of the sweep tone Fourier series for high deviation ratios.

as the phase projection in Fig. 9. The desired sideband amplitude is the length of segment joining the appropriate points on the Cornu spiral. It should also be pointed out that for high indexes the spectral distribution will approach the amplitude probability distribution of the modulating waveform. For a perfect sawtooth this will be a square spectrum with the characteristic frequency terminating overshoot analogous to Gibb's phenomenon, while for a slightly exponential sawtooth the spectrum will appear to be tilted.

Several important points may now be established concerning this forcing function. First, the spectrum is quite well behaved for all modulation indexes. As one starts with an unmodulated carrier and begins modulation at progressively higher indexes, it will be noted that there is no carrier or sideband nulling as is the case with sinusoidal frequency modulation. Thus any deviation ratio is valid for time-delay spectrometry. Second, the spectral sidebands are down 6 dB at the deviation band-edge and are falling off at the rapid rate of about 10 dB per unit analyzer bandwidth. Thus the spectrum is quite confined and no unusual system bandwidth is required. Third, an expansion of the negative frequencies will show a comparable symmetric spectrum, which means that spectrum foldover effects such as those due to passing through zero beat do not in any way compromise the use of this type of modulation for time-delay spectrometry.

A final point in this spectral analysis: one tends to take for granted the validity of the analytical result without much consideration for the physical mechanism. Equation (16) shows that this glide tone can be generated by connecting a large number of signal generators with almost identical amplitude to a common summing junction. Furthermore, the amplitude of the resultant glide tone will be equal to that of any one of the separate generators. At first glance this would not only seem to violate conservation of energy but it would definitely be hard to convince an observer that when he heard a smooth glide tone with definite pitch as a function of time he was actually hearing all possible frequencies. Inspection of the phases as controlled by the partial Fresnel integrals reveals that at any given time all generators except those in the vicinity of the instantaneous glide tone frequency will cancel to zero, and that the distribution of those generators which are not cancelled is approximately Gaussian about the perceived tone. In addition, the "bandwidth" of this distribution is of the order of the analyzer bandwidth. The result of this is that an analyzer slightly detuned from the glide tone will not only be down in response but will be down by the same amount at all frequencies.

Spectrum Analyzer Response

In considering the nature of the display presented by a tracking spectrum analyzer used for time-delay spectrometry, the generalized circuit of Fig. 10 will be used. The portion within the dashed contour characterizes the general spectrum analyzer. The sweeping local oscillator signal is brought out and mixed in a balanced modulator with a fixed local oscillator which has a slight offset from the analyzer intermediate frequency. The filtered difference output of the mixing process is the audio glide tone previously discussed, which is used as a forcing function for the acoustical system under analysis. The acoustical response of this system as perceived by a microphone will consist of a multiplicity of time-delayed responses. The entire signal from the microphone is sent to the spectrum analyzer. The purpose of the analysis below is to demonstrate that any given delayed response may be selected from all inputs by appropriate selection of the glide tone generating offset oscillator, and to de-

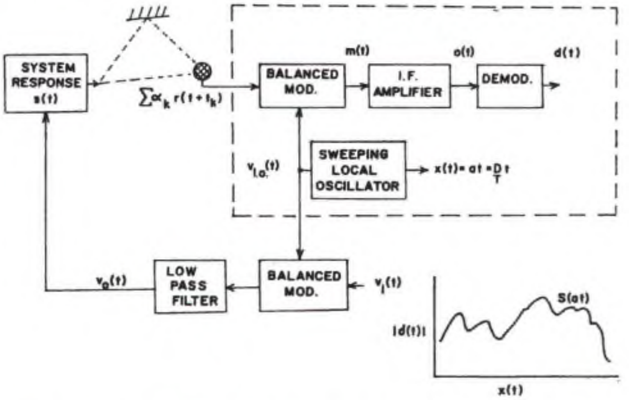


Fig. 10. Generalized block diagram of the acoustical system test using time-delay spectrometry.

velop conditions and limitations of analyzer display.

The analytical description of a sweeping tone used as a system-forcing function leads to a remarkable property: The sweep tone is the complex conjugate of its own Fourier Transform, with time in place of frequency. Just as a steady sinusoid applied to a network provides a narrow frequency window which pulls out the frequency response of the network, the sweep tone may be shown to provide a spectrum window pulling out the frequency spectrum of the network. As a consequence of this property this sweep function $w(t)$ will be designated as a window function,

$$w(t) \triangleq e^{i\frac{1}{2}at^2}, \qquad (22)$$

where a represents the angular dispersion rate D/T.

Proceeding to the analysis it will be observed that the analyzer local oscillator consists of a sweeping tone $w(t)$ heterodyned to the intermediate frequency ω_i. It is a real time function and is thus

$$v_{1.0}(t) = [w(t)e^{i\omega_i t} + w^*(t)e^{-i\omega_i t}]/2 \qquad (23)$$

where the starred operation indicates complex conjugation. Similarly, the down-converting oscillator consists of the intermediate frequency ω_i offset by a fixed value ω_o and is

$$v_1(t) = [e^{i(\omega_i+\omega_o)t} + e^{-i(\omega_i+\omega_o)t}]/2. \qquad (24)$$

After low-pass filtering, the system-driving function, neglecting constant gain terms, is

$$v_0(t) = w(t)e^{-i\omega_o t} + w^*(t)e^{+i\omega_o t}. \qquad (25)$$

Assuming the system response is linear, the system response $r(t)$ for this driving function is the convolution integral of the driving function $v_0(t)$ and the system time response $s(t)$:

$$r(t) = \int_{-\infty}^{\infty} s(\tau)v_0(t-\tau)d\tau \triangleq s(t) \otimes v_0(t). \qquad (26)$$

The second symbolism will be utilized because of its simplified form.

Because of the finite time delay, t_k, between system output and microphone input for each probable path K, the microphone response $r(t)$ consists of the sum of all the signal path inputs,

$$r(t) = \sum_K a_K r(t+t_k) \qquad (27)$$

where a_K is the strength of the K^{th} signal. Also, each separate path length signal is expressible as

$$r(t+t_k) = [s(t+t_k) \otimes w(t+t_k)e^{-i\omega_o(t+t_k)}] + [s(t+t_k) \otimes w^*(t+t_k)e^{i\omega_o(t+t_k)}]. \qquad (28)$$

The process of balanced modulation is a simple multiplication in the time domain. The modulator output $m(t)$ is then, neglecting constant multipliers,

$$m(t) = \sum_K a_K r(t+t_k)\cdot(w(t)e^{i\omega_i t} + w^*(t)e^{-i\omega_i t}). \qquad (29)$$

The frequency spectrum of the modulator output $M(\omega)$, considering only positive frequencies, is

$$M(\omega) = \sum_K a_K \int e^{-i(\omega-\omega_i)t} w(t)r(t+t_k)dt. \qquad (30)$$

(The convention followed for the purpose of this analysis is that lower-case functional notation designates time dependence while upper-case notation designates frequency dependence.)

From Eqs. (A5), (A6), and (A9) of the Appendix, Eq. (30) may be rewritten as

$$M(\omega) = \sum_K a_k \int e^{-i(\omega-\omega_i+at_k-\omega_o)t} W(at_k-\omega_o)(a/2\pi i) \{W(\omega_o)w(\xi)[s(\xi)\otimes w(\xi)] + W^*(\omega_o)w(\xi)[s(\xi)\otimes w^*(\xi)]\}dt \qquad (31)$$

where $\xi = t+t_k-(\omega_o/a)$.

From Eqs. (A7) and (A8) in the Appendix, we may expand this into two integral summations

$$M(\omega) = \sum_K a_K \int e^{-i(\omega-\omega_i+at_k-\omega_o)t} W(at_k-\omega_o)(a/2\pi i)^{3/2} \{W(\omega_o)w(\xi)w(\xi)[S(a\xi)\otimes W(a\xi)]\}dt + \sum_K a_K \int e^{-i(\omega-\omega_i+at_k-\omega_o)t} W(at_k-\omega_o)(a/2\pi i)^{3/2} \{W^*(\omega_o)[S(a\xi)\otimes W^*(a\xi)]\}dt. \qquad (32)$$

The first integral is the balanced modulator "upper sideband" and is the transform of a scanned time spectrum multiplied by two window functions. The transform will therefore be in the vicinity of the intermediate frequency only for $\xi = O$ and will constantly retreat from ω_i for all other times at a rate twice that of the sweeping local oscillator. As a consequence, this integral need not be considered further.

The second integral is the "lower sideband" and will transform the scanned spectrum to lie directly on the frequency $(\omega_i+\omega_o-at_k)$. For both integrals the "bandwidth" of any substantial energy to be found in the transformed spectrum will be determined by the convolution of $W(at)$ and $W^*(at)$ respectively with the system frequency spectrum, with the frequency parameter replaced by time. Figure 11 symbolizes the spectral energy distribution given by Eq. (32).

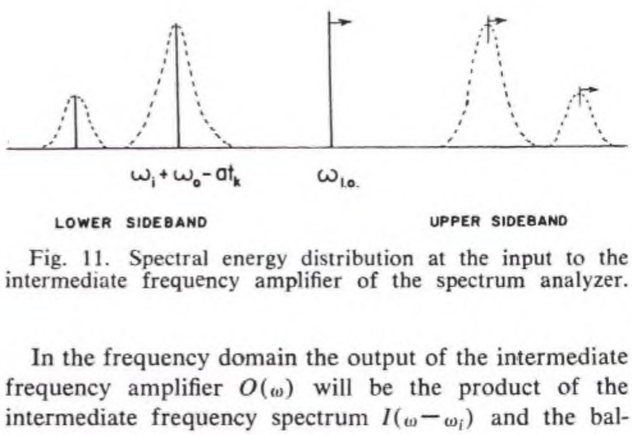


Fig. 11. Spectral energy distribution at the input to the intermediate frequency amplifier of the spectrum analyzer.

In the frequency domain the output of the intermediate frequency amplifier $O(\omega)$ will be the product of the intermediate frequency spectrum $I(\omega-\omega_i)$ and the balanced mixer spectrum:

$$O(\omega) = \sum_K a_k I(\omega-\omega_i)M(\omega-\omega_i+at_k-\omega_o). \qquad (33)$$

If the delay times t_k are not so close together as to overlap the functions $M(\omega-\omega_i+at_k-\omega_o)$ in the vicinity of $(\omega-\omega_i)$, then any particular response may be selected by adjusting ω_o such that

$$\omega_o = at_k \qquad (34)$$

for the desired path time delay t_k. Then Eq. (33) becomes

$$O(\omega) = G_K I(\omega-\omega_i)M(\omega-\omega_i). \qquad (35)$$

The time spectrum of the output of the intermediate frequency amplifier is thus

$$o(t) = g_k[i(t)\otimes W^*(at)\otimes S(at)](a/2\pi i)^{3/2} \qquad (36)$$

or, noting Eq. (A3) in the Appendix and regrouping in accordance with the associative property of convolution,

$$o(t) = g_k(a/2\pi i)\,[i(t) \otimes w(t)] \otimes S(at). \qquad (37)$$

The demodulator output $d(t)$ is the signal displayed on the vertical axis and is thus

$$d(t) = (constant) \cdot |o(t)|. \qquad (38)$$

Inspection of Eqs. (37) and (38) discloses that the proper spectrum has indeed been displayed. It should be recognized that while the analysis was done on a continuous basis no loss of validity is experienced for repetitive sweeps.

In interpreting Eq. (37) it is apparent that the proper spectrum is modified by the window function and time response of the intermediate-frequency amplifier of the analyzer. The action of these latter terms is a smoothing upon the actual spectrum. This smoothing is similar in character to a simple low-pass filtering of the perfect spectrum.

To demonstrate that no spectrum bias is introduced in Eq. (37) by $w(t)$ and $i(t)$, consider the case of a perfectly flat spectrum where

$$S(at) = Constant \qquad (39)$$

and where a finite time is assumed. In this case the convolution integrals collapse to simple integrals yielding

$$o(t) = Constant. \qquad (40)$$

If, in other words, the system spectrum is independent of frequency, the spectrum analyzer display will show a straight line whose height above the baseline will be proportional to the gain or loss through the system.

Looking at the other end of the functional dependence, consider the case of a system spectrum response which is zero for all frequencies but one, and is infinite at that singular frequency. What, in other words, is the response to a network of infinite Q. Such a function is a Dirac delta

$$S(at) = \delta(t - t_o) \qquad (41)$$

simple substitution into Eq. (37) yields

$$o(t) = K[i(t) \otimes w(t - t_o)]. \qquad (42)$$

This is exactly the same response as experienced by a spectrum analyzer viewing a single sinewave spectrum.[4] This zero width spectrum is displayed as a smoothed function, closely approximating the intermediate frequency response for sweep rates such that

$$B^2 \geqq dF/dt. \qquad (43)$$

As the sweep rate increases above the inequality of Eq. (43), the displayed function broadens and the "phase tail" of $w(t) \otimes i(t)$ evidences an increasing ring on the trailing edge. Thus for the normal spectrum analyzer the infinite Q response is smoothed to the order of the analyzer bandwidth.

One exceedingly important result follows from Eq. (37). This is that the spectrum is preserved not only in amplitude but also in phase. Thus the actual spectrum experiences both an amplitude and a phase smoothing. This gives us a measurement tool which could not have been predicted from the intuitive reasoning utilized. The implications are significant, since it is now possible to isolate the influence of an acoustic subsystem without removing that subsystem from its natural environment and analyze complex behavior, amplitude, and phase, in response to an applied stimulus.

CONCLUSION

What has been described is an acoustical testing procedure which allows selective spatial probing of a natural environment by commercially available equipment. The results are displayed immediately in the form of pressure response as a function of frequency, and require no further processing for interpretation. There are of course resolution limitations imposed by the dimensions of the testing area, but these are easily calculated and are small penalties to pay for the privilege of making formerly unobtainable on-location tests.

An acoustic model of reflecting objects has been introduced which identifies the object with an equivalent frequency response expressible as a frequency-dependent scattering coefficient. This frequency response modifies the frequency response of a sound source in a manner analogous to its electronic circuit counterpart, but possesses a time delay corresponding to its position relative to source and auditor. The effect of an assemblage of reflecting objects can be represented as a three-dimensional plot of pressure, frequency and time.

The utilization of time-delay spectrometry is limited primarily by the imagination and ingenuity of the experimenter. The work described in this paper attempted to yield detailed insight into the technique yet not prevent its application because specialized equipment was unavailable. It must be clear that there is still a substantial class of meaningful measurements which not only require extensive modification of the equipment but also demand further understanding and analysis, in particular the phase angle which is analytically available as well as the amplitude of response. Electronic circuit concepts were used to develop the technique as well as the acoustic model. Perhaps once we possess detailed complex spectra of various acoustic systems we may be able to utilize the existing wealth of circuit techniques to reduce the multi-dimensional real-world acoustic problems to a form more amenable to analysis. It may well be that time-delay spectrometry is a tool which will contribute to this goal.

APPENDIX

Summary of Important Relations

1. $f(t) = (1/2\pi) \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega,$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

2. $w(t) = e^{i\frac{1}{2}at^2} \qquad W(\omega) = (2\pi i/a)^{\frac{1}{2}} e^{-i(\omega^2\, 2a)}$

3. $w(t) = (a/2\pi i)^{\frac{1}{2}} W^*(at)$

4. $\int_{-\infty}^{\infty} f(t) g(x-t) dt \triangleq f(x) \otimes g(x)$

5. $s(t+t_k) \otimes w(t+t_k)\, e^{-i\omega_o(t+t_k)} =$
$$(a/2\pi i)^{1/2}\, W(\omega_o)\,[s(t+t_k-\omega_o/a) \otimes w(t+t_k-\omega_o/a)]$$

6. $s(t+t_k) \otimes w^*(t+t_k)\, e^{i\omega_o(t+t_k)} =$
$$(a/2\pi i)^{1/2}\, W^*(\omega_o)\,[s(t+t_k-\omega_o/a) \otimes w^*(t+t_k-\omega_o/a)]$$

7. $w(t)\,[s(t) \otimes w(t)] =$
$$w(t)\,w(t)\,[S(at) \otimes W(at)]\,(a/2\pi i)^{1/2}$$

8. $w(t)\,[s(t) \otimes w^*(t)] = [S(at) \otimes W^*(at)]\,(a/2\pi i)^{1/2}$

9. $w(t) =$
$$w(t+t_k-\omega_o/a)\,W(at_k-\omega_o)\,e^{-i(at_k-\omega_o)t}\,(a/2\pi i)^{1/2}$$

REFERENCES

1. W. B. Snow, "Loudspeaker Testing in Rooms," *J. Audio Eng. Soc.* **9**, 54 (1961).

2. D. H. Bastin, "Microphone Calibrator," *Electronics* (Nov. 1948).

3. T. J. Schultz and B. G. Watters, "Propagation of Sound Across Audience Seating," *J. Acoust. Soc. Am.* **36**, 885 (1964).

4. H. W. Batten, R. A. Jorgensen, A. B. MacNee and W. W. Peterson, "The Response of a Panoramic Analyzer to C.W. and Pulse Signals," *Proc. I.R.E.*, 948 (1954).

5. G. E. Peterson, G. A. Hellwarth, and H. K. Dunn, "An Anechoic Chamber with Blanket Wedge Construction," *J. Audio Eng. Soc.* **15**, 67 (1967).

6. For example, E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945).

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